Nonlinear Magnetoplasmons in Strongly Coupled Yukawa Plasmas

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The existence of plasma oscillations at multiples of the magnetoplasmon frequency in a strongly coupled two-dimensional magnetized Yukawa plasma is reported, based on extensive molecular dynamics simulations. These modes are the analogues of Bernstein modes which are renormalized by strong interparticle correlations. Their properties are theoretically explained by a dielectric function incorporating the combined effect of a magnetic field, strong correlations and finite temperature.

Two-dimensional (2D) systems with strong correlations, i.e., with the interaction energy exceeding the thermal energy, show a number of unusual properties ranging from anomalous phase transitions (Kosterlitz-Thouless scenario) to anomalous transport [1,2]. Examples include electrons on helium droplets and in quantum dots [3], ions in traps [4], and dusty plasmas [5–7]. A particular correlation effect is observed in the collective oscillation spectrum of 2D Coulomb and Yukawa liquids [5,8,9] where a transverse shear mode has been predicted by Kalman and Golden [10,11]. This mode does not exist in an ideal system but has experimentally been observed in strongly coupled systems [12,13]. Similarly, in the presence of a strong magnetic field a 2D plasma shows two coupled modes—a magnetoplasmon and a magentoshear mode (upper and lower hybrid modes), which have recently been studied in detail in the liquid and crystal phases [6,8,9].

Here we show that a strongly correlated 2D Yukawa plasma (2DYP) possesses additional collective modes. We demonstrate that these are related to Bernstein modes (BMs) [14] which are well known in high-temperature classical [15,16] and quantum plasmas [17,18]. However, these are all nearly ideal plasmas. Here we demonstrate that such modes can also exist in strongly correlated systems, but they have a number of peculiar properties arising from the combined effect of correlations and the magnetic field.

Model and simulation method.—Our 2DYP consists of N particles in a quadratic monolayer (with periodic boundary conditions), subject to a magnetic field perpendicular to the plane and interacting by Yukawa forces. The coupled Newton's equations are ($i = 1, \ldots, N$)

$$m\ddot{\mathbf{r}}_i = \frac{Q}{c}\dot{\mathbf{r}}_i \times \mathbf{B} - Q^2\sum_{j\neq i}\left(\nabla\frac{e^{-r/\lambda_D}}{r}\right)\bigg|_{\mathbf{r}=\mathbf{r}_i-\mathbf{r}_j}, \quad (1)$$

with the Debye screening length λ_D and charge Q. Here we define the Wigner-Seitz radius $a = (n_0\pi)^{-1/2}$ (where n_0 is the areal density) and the frequency $\omega_0 =$ $(2\pi Q^2 n_0/ma)^{1/2}$, to be distinguished from the k-dependent 2D analogon of the plasma frequency, $\omega_p(k) = [2\pi Q^2 n_0 k^2/(m\sqrt{k^2+\kappa^2})]^{1/2}$. In thermodynamic equilibrium, the system is fully characterized by three parameters—the dimensionless inverse screening length $\kappa = a/\lambda_D$, the Coulomb coupling parameter $\Gamma = Q^2/(ak_BT)$, and the dimensionless magnetic field strength $\beta = \omega_c/\omega_0$, with the cyclotron frequency $\omega_c = QB/mc$. Equation (1) is solved by a microcanonical molecular dynamics (MD) method which incorporates an arbitrarily strong static homogeneous magnetic field into the second-order velocity Verlet algorithm [19]. Data acquisition starts after the system has equilibrated to the desired temperature (Γ). The longitudinal and transverse collective excitation spectra, $L(k, \omega)$ and $T(k, \omega)$, respectively, are computed from the microscopic longitudinal and transverse current fluctuations [20]. Here we use $N = 4080$ particles and cover a broad parameter range with $1 \leq \kappa \leq 3$, $10 \leq \Gamma \leq 1000$, and $0.01 \leq \beta \leq 1.4$.

Numerical results.—A first overview of the simulation results is given in Fig. 1 showing the evolution of the collective oscillation spectrum with the coupling parameter Γ at a fixed field strength $\beta = 0.5$. At low wave numbers k, one clearly observes two peaks, one of which starts at $\omega = \omega_c = 0.5$ and a second at very low frequency. When Γ is increased [cf. Figs. 1(a)–1(c)], the peaks become more pronounced and extend to higher k. These are the well-known magnetoplasmon (MP) and magnetoshear modes—the latter being a pure correlation effect as noted above [10]. When the coupling is increased beyond the crystallization point (when the system transforms into a microcrystalline structure in our simulation) [cf. Fig. 1(d)], the modes transform into the magnetophonon spectrum [6,9]. The properties of these two modes are well described within the quasilocalized charge approximation (QLCA) and harmonic lattice theory and very well agree with MD simulations; thus it appears that the collective excitations of strongly correlated Coulomb and Yukawa systems are fully understood.
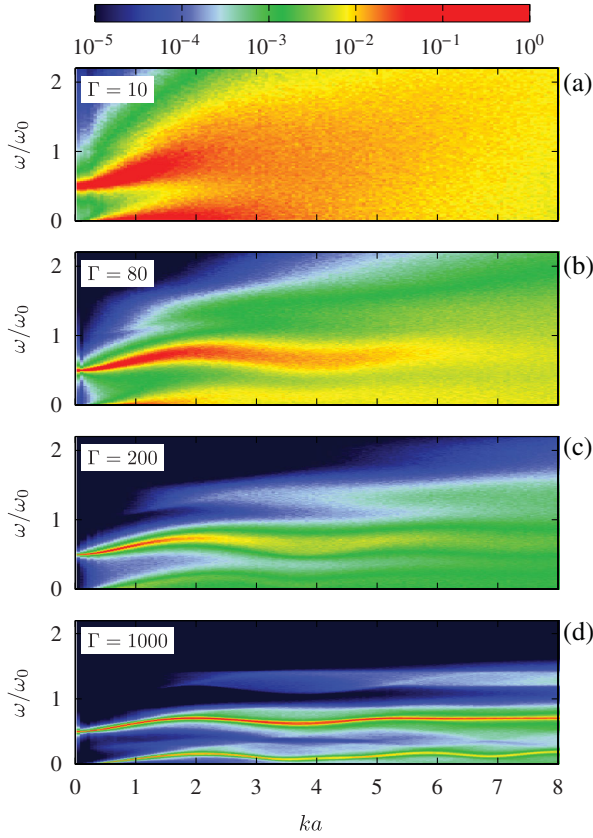
FIG. 1 (color). Collective excitation spectrum $L(k, \omega) + T(k, \omega)$ of a Yukawa plasma with $\kappa = 2$ and $\beta = 0.5$, for four values of the coupling parameter given in the figure. The amplitudes of the spectra are normalized by their peak values.

FIG. 2 (color online). (a),(c),(d) Longitudinal magneto-oscillation spectrum $L(k, \omega)$ of a Yukawa plasma with $\kappa = 2$ for various field strengths and coupling parameters. (b) Transverse spectrum $T(k, \omega)$ for the parameters of (a). The lines are vertically offset for better visibility.

However, this is not the case. Let us look at frequencies above the MP. For $\Gamma = 80$ [Fig. 1(b)], a third maximum appears in the spectrum around $1.3\omega_0$, starting at a wave number of about $ka = 1.5$. This peak is very broad and strongly overlaps with the MP. An analysis of the spectrum for $\Gamma = 200$ and 1000 reveals that this peak exists there as well—it is now more clearly separated from the MP; cf. Figs. 1(c) and 1(d). To explore the origin of this peak, we have performed a series of simulations for a broad range of field strengths, coupling parameters, and screening parameters. All the results confirm the appearance of not just one peak but of a whole series of additional collective modes. Some representative results are collected in Figs. 2 and 3. Both figures clearly show up to four additional plasmon modes, which are equally spaced, where the spacing increases with B.

These additional peaks and their monotonic dependence on the cyclotron frequency strongly resemble Bernstein modes [14]. Furthermore, the coincidence of longitudinal and transverse current fluctuation spectra (cf. Fig. 2, top row) clearly confirms the circular polarization. In fact, electrostatic modes with frequencies around multiples of ω_c are well known in many *ideal* classical and quantum plasmas; see above. But here they are observed, for the first
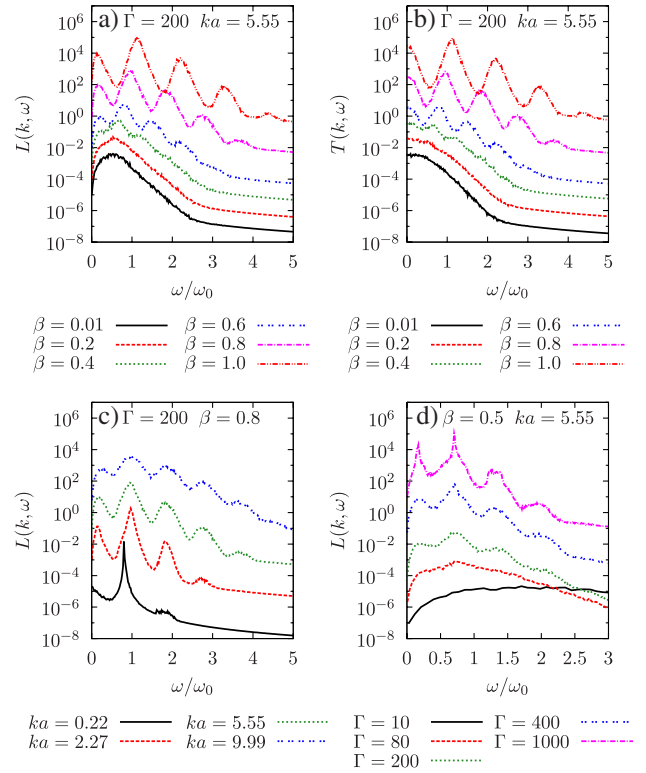
time, in a *strongly coupled system*. Note that neither cold fluid theory nor the QLCA have revealed such modes because they neglect finite temperature effects, which are crucial for the excitation of the harmonics; see below. We, therefore, develop a kinetic theory of a magnetized plasma at finite temperature which incorporates strong correlations.

Theory.—We start from the retarded longitudinal dielectric function ϵ^l of a one-component correlated plasma in a magnetic field [21]:

$$\epsilon^l(\omega, k) = \frac{k_i k_j}{k^2} \epsilon_{ij}(\omega, k) = 1 - V(k)\Pi^l(\omega, k), \quad (2)$$

where ϵ_{ij} is the dielectric tensor and $V(k) = 2\pi Q^2/(k^2 + \kappa^2)^{1/2}$ is the Fourier transform of the 2D Yukawa potential (our results include the Coulomb case, for $\kappa \rightarrow 0$). The longitudinal polarization Π^l contains all interaction effects and can be expressed through the uncorrelated (Vlasov or RPA) polarization Π^l_0 via the Bethe-Salpeter equation [22] or via

$$\Pi^l(\omega, k; B) = \frac{\Pi^l_0(\omega, k; B)}{1 + V(k)\Pi^l_0(\omega, k; B)G(k, \omega; B)}, \quad (3)$$

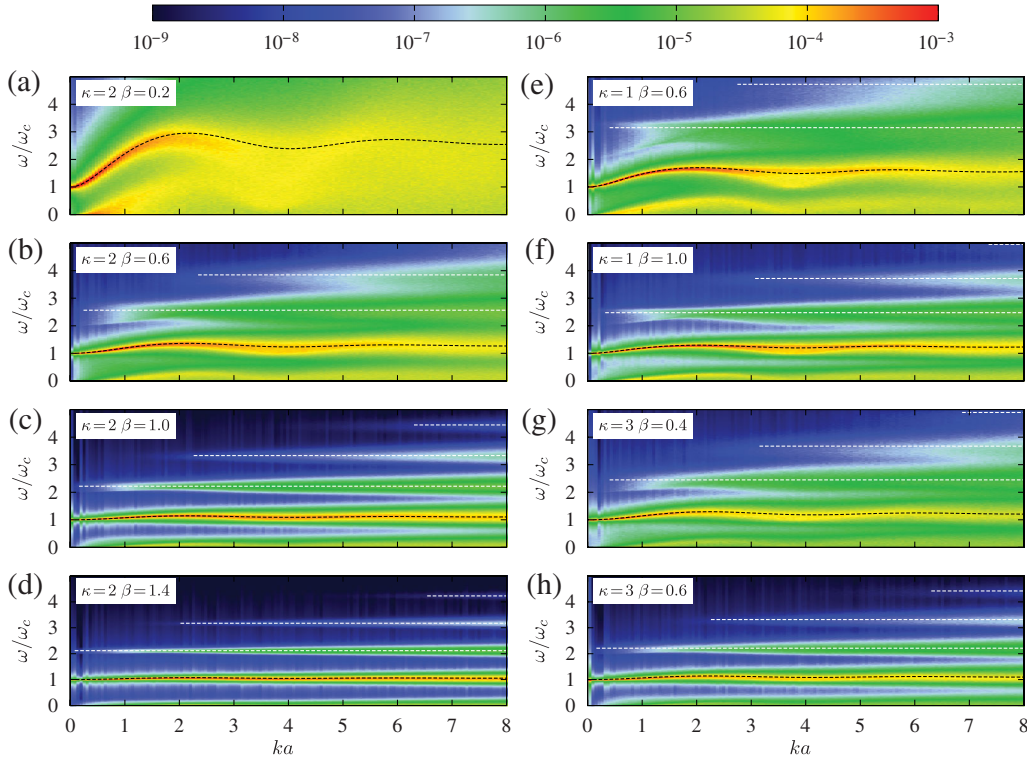where $G(k, \omega; B)$ is the so-called local field correction [23].

FIG. 3 (color).   MD collective excitation spectra $L(k, \omega) + T(k, \omega)$ (color) vs theoretical dispersions [Eqs. (6) and (8)] based on the corrected polarization $\tilde{\Pi}_0^l$ (black and white lines). Left column: $\Gamma = 200$; right column: $\Gamma = 150$ (upper two figures) and $\Gamma = 600$ (lower two figures). Note that frequencies are scaled in units of ω_c.

However, G is known only approximately; results for the electron gas have been given, e.g., by Singwi *et al.* [23]. For a strongly correlated plasma, Kalman and Golden observed [11] that the collective modes in the cold fluid description are well reproduced by a static approximation of (3) in which the magnetic field is neglected: $G(k, \omega; B) \approx -D(k)/\omega_p^2(k)$, where D is the dynamical matrix of QLCA (a functional of the pair distribution function) [10]. Here we extend this concept to finite temperatures and to the BMs. To this end we compute the Vlasov polarization $\Pi_0^l(\omega, k; B)$ of a 2D Maxwellian plasma with Yukawa interaction in a magnetic field, which is analogous to the quantum RPA result [17]:

$$\Pi_0^l(\omega, k; B) = \frac{2n_0}{k_B T} e^{-z} \sum_{n=1}^{\infty} \frac{n^2 \omega_c^2}{\omega^2 - (n\omega_c)^2} I_n(z), \quad (4)$$

where I_n denotes the modified Bessel function, $z = (k v_T/\omega_c)^2$, and $v_T^2 = k_B T/m$ is the square of the thermal velocity. By using this result and Eqs. (2) and (3), the dispersion relation of the longitudinal modes is obtained from $\epsilon^l(\omega, k; B) = 0$:

$$0 = 1 - 2\tilde{\omega}_p^2(k) \frac{e^{-z}}{z} \sum_{n=1}^{\infty} \frac{n^2}{\omega^2 - (n\omega_c)^2} I_n(z), \quad (5)$$

where $\tilde{\omega}_p^2(k) = \omega_p^2(k) + D(k)$, and we use the approxima-

tion $D(k) = D_L(k) + D_T(k)$. Equation (5) can be further simplified by noticing that $z = (ka)^2/(2\beta^2\Gamma) \ll 1$ for the situations of interest. For example, for $\beta = 1$ and $\Gamma = 200$, as in Fig. 3(c), $z \lesssim 0.16$, up to $ka = 8$. Then, by using $I_n(z) \approx (z/2)^n/n!$, Eq. (5) yields the coupled MP and BMs, which are modified by correlations. Approximately we find, for the MP, the well-known dispersion

$$\omega_1^2(k) \approx \omega_c^2 + \tilde{\omega}_p^2(k), \quad (6)$$

and, additionally, for the BMs ($n \geq 2$), the frequencies

$$\omega_n^2(k) \approx (n\omega_c)^2 + [n\tilde{\omega}_p(k)]^2 \frac{1}{n!}\left(\frac{z}{2}\right)^{n-1}. \quad (7)$$

The latter are found to be undamped and to exist in the entire wave number range with frequencies rather close to the simulation results (note that since $z \ll 1$ the second contribution to ω_n is negligible for the shown parameters); cf. Figs. 3(d) and 3(h). This clearly confirms our interpretation in terms of Bernstein modes.

However, a closer inspection of the simulation data reveals three qualitative differences: (i) All BMs are damped; (ii) they exist only beyond a finite wave number $k_{cr}^{(n)}$ which increases with n; (iii) the BMs *are not spaced by* ω_c but by a different frequency; cf. Figs. 3(b) and 3(f). In fact, we find that the frequencies are very well described by

$$\omega_n^2(k) \approx n^2 \omega_{1,\infty}^2, \qquad \omega_{1,\infty}^2 = \omega_c^2 + 2\omega_E^2, \qquad (8)$$

where $\omega_{1,\infty}$ is the large k asymptotic of the MP (6) and ω_E is the Einstein frequency describing the average oscillation frequency of a test particle in the frozen environment of the other particles. The surprising absence of any wave number dispersion in the numerical results (cf. Figs. 2 and 3) suggests that BM excitation is governed by processes on the smallest length scale of single particles. In fact, in a strongly correlated plasma, particles are trapped in instantaneous local potential minima created by all other particles [24]. The trajectories of these randomly oscillating particles are ''bent'' by the Lorentz force onto cyclotron orbits around the magnetic field direction giving rise to an average gyration frequency $\omega_{1,\infty}$. In the presence of thermal fluctuations, nonlinear energy exchange with short wavelength magnetoplasmons leads to particle gyration with frequencies $n\omega_{1,\infty}$, connected with plasmon emission of these harmonics. Thus, the observed oscillations are ''dressed Bernstein modes'' arising from a combined effect of magnetic field and strong correlations which increases the fundamental frequency from ω_c to $\omega_{1,\infty}$.

It is straightforward to present a corrected polarization $\tilde{\Pi}_0^l(\hat{\omega}, k; B)$ which reproduces all the above effects. The main difference, compared to Eq. (4), is that the denominators of the terms $n \geq 2$ are of the form [25] $\hat{\omega}\omega - (n\omega_{1,\infty})^2$, which yields the correct BM dispersion (8). Further, by introducing a phenomenological damping constant $\nu > 0$, with $\hat{\omega} = \omega + i\nu$, it is possible to correctly reproduce the damping of the BMs and the finite k values at which they emerge. This damping emulates collisional dissipation effects, which are missing in the Vlasov theory and in the QLCA but are present in the simulations. Replacing, in the dispersion relation (5), $\Pi_0^l(\omega, k; B)$ by $\tilde{\Pi}_0^l(\hat{\omega}, k; B)$ yields, for the critical wave number of the nth BM, $k_{cr}^{(n)} a = 2\beta\Gamma^{1/2}[(n-1)! \omega_{1,\infty}\nu/\omega_E^2]^{1/(2n-2)}$. Using ν as a free parameter allows us to reproduce the finite values $k_{cr}^{(n)}$ and their increase with n observed in the simulations, as is shown by the dashed white lines in Fig. 3. We underline that this agreement is representative for magnetized 2DYP in the strongly coupled liquid and crystal phases, which is supported by additional results for different screenings; see the right column of Fig. 3.

In summary, we have demonstrated that a strongly correlated 2DYP possesses, in addition to the well-known magnetoplasmon and magnetoshear modes, a sequence of Bernstein-type modes. In contrast to the familiar situation of weakly nonideal high-temperature classical and low-temperature quantum plasmas where the BMs appear at multiples of ω_c, in a correlated plasma the modes are renormalized by correlations and appear at harmonics of the correlated MP frequency $(\omega_c^2 + 2\omega_E^2)^{1/2}$. It is the combined effect of correlations and magnetic field which yields the strong harmonics signal with the frequencies (8) reported in this Letter [26]. The predicted dressed Bernstein

modes should be observable, for example, in trapped ions, nonideal electron-hole plasmas in quantum wells, and dusty plasmas. Using, e.g., dust particles of 0.5 μm diameter and a magnetic field of 20 T gives rise to a strongly coupled weakly magnetized liquid state with $\Gamma \sim 100$ and $\beta \sim 0.1$, which should allow for resonant detection of mode $n = 2$.

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