Contrib. Plasma Phys. **52**, No. 1, 70 – 73 (2012) / **DOI** 10.1002/ctpp.201100096

# Instabilities in Yukawa Liquids

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Received 18 October 2011, revised 28 November 2011, accepted 28 November 2011
Published online 26 January 2012

**Key words** Dusty plasma, Yukawa liquid, instabilities.

A strongly coupled Yukawa liquid is a system of charged particles which interact via a screened Coulomb interaction and in which the electrostatic energy between neighboring particles is larger than their thermal energy but not large enough for crystallization. Various plasma systems including ultracold neutral plasmas and complex (dusty) plasmas can exist in this strongly coupled liquid phase.Here we investigate instabilities driven by the relative streaming of plasma components in three-dimensional Yukawa liquids with a focus on complex plasmas. This includes a dust acoustic instability driven by weakly coupled ions streaming through the dust liquid, and a dust-dust instability driven by the counter-streaming of strongly coupled dust grains. Compared to the Vlasov behavior we find there can be a substantial modification of the unstable wavenumber spectrum due to strong coupling effects.



## 1 Introduction

A Yukawa system in which the charged particles interact via a screened Coulomb interaction is characterized by two parameters, $\Gamma = q^2/aT$ and $\kappa = a/\lambda_D$. Here $q$ is the particle charge, $T$ is the thermal (kinetic) energy of the particles, $a$ is the average interparticle distance, and $\lambda_D$ is the screening length in the background medium. The liquid phase corresponds to an effective screened Coulomb coupling parameter, $\Gamma e^{-\kappa}$ (a more precise value of the effective coupling is given in [1]), which is $\gg 1$ but smaller than that required for crystallization ($\sim 170$ for a three-dimensional (3D) system [2]). Various plasma systems including complex (dusty) plasmas [3, 4] or ultracold neutral plasmas [5] can exist in this strongly coupled liquid phase. Strong coupling affects the dispersion of collective modes in these plasmas [6, 7, 8, 9], particularly in the finite wavenumber, $k$, regime where the modes can exhibit negative dispersion. It is expected that the behavior of instabilities should also be affected by strong coupling [10, 11, 12]. Here we investigate instabilities driven by the relative streaming of plasma components in 3D Yukawa liquids with a focus on dusty plasmas. The quasi-localized charge approximation is used to describe the dynamical behavior of the strongly coupled component (see [6, 13]). We consider (a) a dust acoustic instability driven by weakly coupled ions streaming through the dust liquid, and (b) a dust-dust instability that may occur when strongly coupled dust grains counter-stream. Compared to the Vlasov behavior we find there can be a substantial modification of the unstable $k$-spectrum induced by strong coupling effects.

## 2 Dust acoustic instability

First we consider a plasma containing electrons, singly charged ions, and strongly coupled negatively charged dust. The condition of charge neutrality in equilibrium is given by $n_e + Zn = n_i$. Here the subscripts $j = e, i$ denote electrons, ions, respectively, while $Z$ and $n$ are the charge state and number density of the dust. We consider the excitation of dust acoustic waves [14] that have phase speed much smaller than the ion and electron thermal speeds, $v_i$ and $v_e$, respectively. The electrons and ions are assumed to stream relative to the dust. If the

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stream speeds are much larger than their respective thermal speeds, a Buneman type instability may occur; this was was considered previously where we found only a small modification at small $k$ [10, 11]. Here we consider a kinetic instability at arbitrary $k$, driven by inverse Landau damping, that can occur when the electron and ion drift speeds $V_{0e}$ and $V_{0i}$ are much smaller than their respective thermal speeds. In a laboratory dusty plasma it is generally the case that the electron temperature $T_e$ is much larger than the ion temperature $T_i$, and this dust acoustic instability is driven primarily by the ions. In this case, the dispersion relation for a one-dimensional dust-acoustic instability, assuming cold dust and neglecting collisional effects, is (see e.g. [11, 15])

$$1 + \frac{1}{k^2\lambda_{De}^2} + \frac{1}{k^2\lambda_{Di}^2}\left[1 + i\sqrt{\frac{\pi}{2}}\frac{(\omega - kV_{0i})}{kv_i}\right] - \frac{\omega_p^2}{\omega^2 - D_L(k)} \approx 0\,. \tag{1}$$

Here $\omega_p = (Z^2e^2 4\pi n/m)^{1/2}$ is the dust plasma frequency where $m$ is the dust mass, and $D_L(k)$ is a local field function characterizing strong coupling effects, which can be numerically computed as a functional of the equilibrium pair correlation function for sets of $\Gamma$ and $\kappa$ which characterize the Yukawa liquid (see [6, 7]). Note that $D_L(k)$ is $< 0$ since it is proportional to the dust correlation energy, and its inclusion modifies the DA wave via a reduction in the acoustic speed and maximum frequency of the wave and the onset of negative dispersion [8, 9].

Assuming $V_{0i} \gg \omega/k$ and taking $\omega = \omega_r + i\gamma$ with $|\gamma| \ll \omega_r$, the solution of (1) is

$$\begin{aligned}\omega_r &\approx \omega_p\left[\frac{k^2\lambda_D^2}{1+k^2\lambda_D^2} + \frac{D_L(k)}{\omega_p^2}\right]^{1/2},\\ \gamma &\approx \sqrt{\frac{\pi}{8}}\frac{k^2\lambda_D^2}{(1+k^2\lambda_D^2)^2}\frac{V_{0i}}{v_i}\frac{\omega_p^2}{\omega_r}\,.\end{aligned} \tag{2}$$

Here, the linearized Debye length $\lambda_D$ is given by $1/\lambda_D^2 = (1/\lambda_{De}^2 + 1/\lambda_{Di}^2)$, where $\lambda_{De}$ and $\lambda_{Di}$ are the electron and ion Debye lengths respectively. (When $T_e \gg T_i$ and $n_i > n_e$ for negatively charged grains, $\lambda_D \approx \lambda_{Di} = (T_i/4\pi ne^2)^{1/2}$. From (2), we see that strong coupling leads to a decrease in the real frequency $\omega_r$ (note $D_L(k) < 0$) so that the growth rate increases as compared with the Vlasov case. This can be seen in Figure 1, which shows the real and imaginary parts of the frequency, $\omega_r$ and $\gamma$, respectively, normalized to $\omega_p$. The solid curves were obtained by solving (1) using numerically computed values for $D_L(k)$ for the case with strong coupling with $\Gamma = 725$ and $\kappa = 3$. For comparison we show the Vlasov case without strong coupling by the dashed curves, which were obtained by solving (1) setting $D_L(k) = 0$. The enhancement of the growth rate may arise because the phase speed of the wave decreases due to strong coupling, so that a larger portion of the ion velocity distribution can participate in the inverse Landau damping mechanism which drives this kinetic instability.

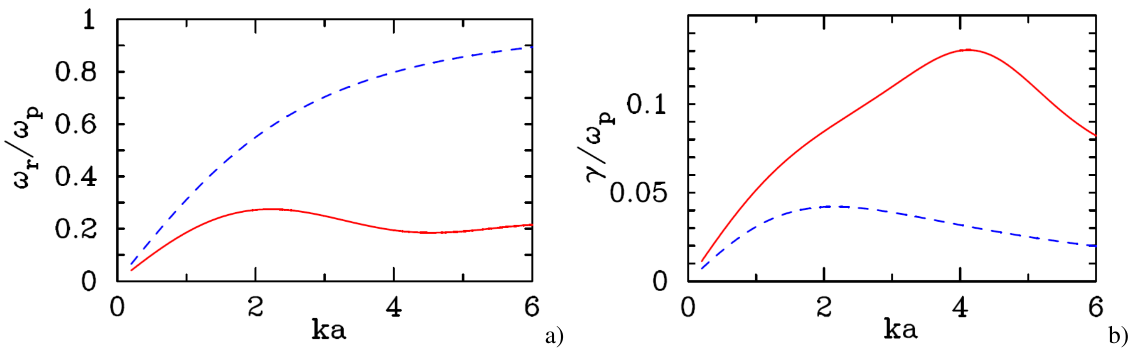


**Fig. 1** a) Real, $\omega_r$ and b) imaginary, $\gamma$ parts of $\omega$ normalized to $\omega_p$ versus $ka$ obtained by solving (1), with $V_{0i} = 0.15\,v_i$ and $T_e = 80\,T_i$. The solid (red) curves correspond to the case with strong coupling with $\Gamma = 725$ and $\kappa = 3$, and the dashed (blue) curves correspond to the case without strong coupling.

## 3 Dust-dust instability

Next we consider a plasma containing electrons, singly charged ions, and two species of negatively charged dust. The condition of charge neutrality in equilibrium is given by $n_e + Z_1 n_1 + Z_2 n_2 = n_i$. Here the subscripts $j = e$, $i$, 1, 2, denote electrons, ions, dust particle species 1, and dust particle species 2, respectively. In addition, $Z_1$, $Z_2$ are the charge states of the dust particle species, and $n_j$ are the number densities of the charged particle species $j$.

We consider a dust-dust instability driven by the counter-streaming of dust particles with stream speed much larger than their respective thermal speeds. In contrast to the kinetic instability discussed in the previous section, this is a hydrodynamic instability; it is a low frequency analog of the ion-ion instability, taking strong coupling into account. The dust particles in each stream are assumed to be strongly coupled in the liquid phase. Strong coupling between the streams is neglected, which may be roughly valid if the Coulomb coupling parameter between the streams is small due to the large relative thermal (kinetic) energy between the streams. We consider wave phase speeds that are much smaller than the ion (and electron) thermal speeds, and assume that the ion and electrons are are Boltzmann distributed.

For simplicity, we consider the symmetric case where the dust species have the same plasma frequency $\omega_p$, are characterized by the same $\Gamma$ and $\kappa$, and stream in opposite directions with speed $u$. Although the asymmetric case would be more realistic, the effect of strong coupling on the instability behavior is similar. We can then describe the one-dimensional dust-dust instability using the fluid dispersion relation

$$1 + \frac{1}{k^2\lambda_D^2} - \frac{\omega_p^2}{(\omega - ku)^2 - D_L(k)} - \frac{\omega_p^2}{(\omega + ku)^2 - D_L(k)} = 0\,. \tag{3}$$

The dispersion relation (3) can be written as

$$1 - \frac{B}{(x - A)^2 - C} - \frac{B}{(x + A)^2 - C} = 0\,, \tag{4}$$

where $x = \omega/\omega_p$, $B = k^2\lambda_D^2/(1 + k^2\lambda_D^2)$, $A = ku/\omega_p$, and $C_L = D_L(k)/\omega_p^2$. Note that $C$ depends on $k$, $\Gamma$ and $\kappa$ and needs to be computed from molecular dynamics simulations. A solution of (4) which can yield a purely imaginary value for $x$ under certain conditions is

$$x = \left[-\sqrt{4A^2C + B^2 + 4A^2B} + C + B + A^2\right]^{1/2}\,. \tag{5}$$

For the Vlasov case, where $C$ = 0, this can yield a purely imaginary value (note Im $x > 0$ implies growth) when $A^2 < 2B$. The latter conditions translates into $u < \sqrt{2}c_s/[1 + k^2\lambda_D^2]^{1/2}$, where $c_s = \lambda_D\omega_p$ is the Vlasov dust acoustic speed. (This result is analogous to the usual instability condition for the ion-ion instability for cold ions, e.g., see [16, 17].) What this condition basically means is that the dust stream speed should be smaller than the phase speed of the dust acoustic wave in the system, in order for the beam line $ku$ to intersect the wave dispersion curve.

The $k$ region of maximum growth is determined by the resonance condition $\omega \sim ku$. Because strong coupling reduces the frequency of the dust acoustic wave (see Fig. 1a), we expect that for a fixed stream speed $u$, the wavenumber $k$ would have to decrease in order for the beam line $ku$ to intersect the dispersion curve of the longitudinal wave in the system. We used computed values for $D_L(k)$ and solved (3) numerically for a fixed value of $u$. The curves in Figure 2 show the imaginary part of $\omega$ as a function of $ka$, obtained by solving (3) for two different speeds, $u$ = 0.6 $c_s$ and $u$ = 1.1 $c_s$. The solid curves show results with strong coupling, for $\Gamma$ = 725 and $\kappa$ = 3 in the liquid phase. (Since this is a symmetric case, the real part of $\omega$ is zero in this system for finite growth rate.) For comparison, the case without strong coupling is obtained by solving (3) with $D_L(k)$ = 0 and is shown by the dashed curves. As can be seen, as compared with the Vlasov case the unstable spectrum shrinks in $k$-space, with the maximum growth rate shifting toward longer wavelengths. In the case of smaller relative drift, with $c_s$ = 0.6, it can be seen that the maximum growth rate is somewhat increased compared to the Vlasov case. This appears to occur at $k$ values where the dispersion relation of the dust acoustic wave can exhibit negative dispersion (see Fig. 1a). This effect may have some relation to prior studies of the effect of strong coupling on the electron beam-plasma instability where it was found that the instability could change from convective to absolute in regions of negative wave dispersion [18].

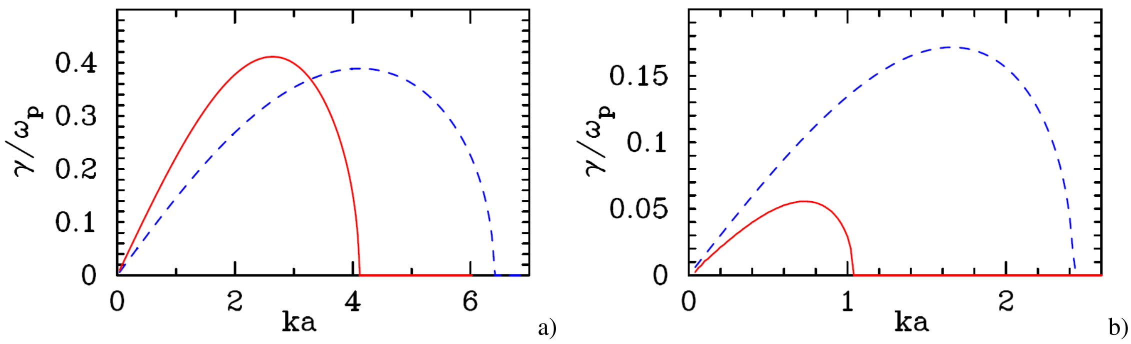


**Fig. 2** Imaginary, $\gamma$ part of $\omega$ normalized to $\omega_p$ versus $ka$ obtained by solving (3). The solid (red) curves correspond to the case with strong coupling with $\Gamma$ = 725 and $\kappa$ = 3, and the dashed (blue) curves correspond to the case without strong coupling. a) $u/c_s$ = 0.6. b) $u/c_s$ = 1.1.

## 4 Summary

We have investigated instabilities driven by the relative streaming of plasma components in 3D Yukawa liquids with a focus on complex (dusty) plasmas. We considered a) a dust acoustic instability driven by weakly coupled ions streaming through the dust liquid, and b) a dust-dust instability driven by the counter-streaming of strongly coupled dust grains. Compared to the Vlasov behavior we find there can be a substantial modification of the unstable spectrum due to strong coupling effects. For the kinetic dust acoustic instability, we find that the growth rate can be enhanced at finite $k$ particularly where the dust acoustic wave exhibits negative dispersion. For the hydrodynamic dust-dust instability we find that for fixed relative dust streaming speed, the unstable $k$-spectrum shrinks and the maximum growth rate shifts to longer wavelengths. It appears that these modifications might be large enough to be observed experimentally, perhaps in future microgravity experiments. Note that even though the dust acoustic instability we have considered develops at fairly high $k$-values, we have ignored Landau damping in the grain system: this is probably justified, since localization associated with strong coupling suppresses Landau damping.

Future work will include the wave damping effect of collisions of background neutrals with charged particles, because dusty plasmas are generally weakly ionized under laboratory conditions. In addition, the issue of possible strong coupling between the dust streams should be considered for the dust-dust instability.

**Acknowledgements** This work was partially supported by NSF grants PHY-0903808, PHY-0813153, PHY-0715227 and PHY-1105005, NASA grant NNX10AR54G, and OTKA PD-75113.

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