Chaotic Current Oscillations with Broadband $1/f^\alpha$ Spectrum in a Glow Discharge Plasma

Z. DONKÓ and L. SZALAI

Research Institute for Solid State Physics of the Hungarian Academy of Sciences, P.O. Box 49, H-1525 Budapest, Hungary

Abstract—We have observed self-generated chaotic current oscillations in the plasma of a helium glow discharge. The $S(f)$ power density spectrum of the current signal exhibited characteristic $S(f) \propto 1/f^\alpha$ dependence over a wide range of frequencies with an exponent of $\alpha = 4.3$. The correlation dimension of the system determined from the correlation sums of the current signal was found to be $D_2 \approx 6.3$. This value of $D_2$ is in good agreement with the theoretically predicted dimension, corresponding to fully developed turbulence. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

During the past decade considerable attention was devoted to the study of physical systems exhibiting chaotic behaviour. The studied dynamical systems in which chaotic oscillations were experimentally observed also included different types of plasmas.

Experiments in the field of gaseous electronics started with the observation of deterministic chaos in the glow discharge plasma of a spectral lamp [1]. Subsequent studies concerned ionization instabilities in the positive column of glow discharges [2, 3], and transition sequences between regular and chaotic oscillations [3, 4]. The existence of self-generated oscillations in a hollow cathode discharge was also demonstrated [5]. The effect of periodic perturbation on helium and neon glow discharges was thoroughly investigated, both experimentally and theoretically [6–8]. Nonlinear behaviour has also been extensively studied in different types of plasma devices (including double plasma devices, plasma reactors, thermionic discharges and fusion devices) [9–17].

It is well known that signals originating from chaotic systems often exhibit $1/f^\alpha$ type frequency spectrum. In the case of plasma chaos, in most papers the measured chaotic signals are reported to have broad power spectrum [2, 3, 6, 8, 10, 14], but the detailed features of the spectrum are not further analysed. Clear evidence of $1/f^\alpha$ type spectrum is given only in a few recent reports [15, 17].

This paper reports an experiment with a positive column dominated glow discharge in low pressure helium gas. We have observed self-generated current oscillations in the discharge plasma of which the power density spectrum exhibits $1/f^\alpha$-like behaviour over a wide frequency range. We have reconstructed phase space trajectories using the time delayed values of the discharge current, and have determined the correlation dimension of the system for a given set of experimental conditions.

EXPERIMENTAL

The experimental setup and the scheme of the discharge tube are shown in Fig. 1. The Pyrex envelope of the tube had an internal diameter of 10.7 mm, the discharge electrodes
had a diameter of 10 mm and they were separated by a distance of 200 mm. A hollow cathode discharge was established by drilling a 7 mm diameter, 20 mm deep hole into the cathode electrode. In this way most of the ion current was concentrated into the inner part of the cathode avoiding the deposition of sputtered cathode material on the wall of the discharge tube. We used spectroscopic quality helium as filling gas.

The discharge was driven by a stable DC voltage source (of voltage $V_{\text{DC}}$) through a 66 kΩ resistor (see Fig. 1). The oscillations of the discharge current were monitored using a 3.3 kΩ resistor to which we have connected an operational amplifier impedance converter. This ensured that the circuit of the discharge tube was not loaded capacitively. The signal was fed through a 10th order low-pass filter (tuned to 90 kHz) and was digitized at a rate of 200 ksamples/sec with a resolution of 12 bits using a Real Time Devices AD3110 analog-digital card.

RESULTS

In our experiment the $I(t)$ current flowing through the discharge circuit was measured. The discharge was started at a low pressure ($p \approx 2$ mbar) where the discharge current was steady and no instabilities were observed. We slowly increased the pressure and observed periodic oscillations. Further increasing the pressure the regular oscillations changed to obviously irregular (chaotic) ones. At this point the gas filling was stopped and the pressure was kept constant during the measurement. The onset of chaotic oscillations occurred at $p = 4.9$ mbar helium pressure at the fixed source voltage $V_{\text{DC}} = 435$ V.

The fluctuations of the current were found to be superimposed on an average value of approximately $\langle I(t) \rangle \equiv 1$ mA. Figure 2(a) displays a typical recorded $I(t)$ current waveform. The current signal plotted in Fig. 2(a) exhibits ‘irregular’ oscillations, although a strong periodic component with a period of $T \approx 0.3$ ms is also present. It is noted that the fluctuations of the current signal are strong, the peak to peak value of the fluctuations is approximately equal to the average current. The $R(\tau)$ autocorrelation function of the current signal was calculated as

$$R(\tau = m\Delta t) = \frac{c}{n} \sum_{k=0}^{n} I'(t_k)I'(t_{k+m}) \quad m = 0, 1, 2 \ldots$$  \hspace{1cm} (1)

where $\Delta t$ is the sampling time, $I'(t_k) = I(t_k) - \langle I(t) \rangle$, $I(t_k)$ is the sample of the current signal taken at time $t_k = k\Delta t$ and $n = N - m$, where $N$ is the length of the data sequence.
(we have usually used 8 kword data series, \( N = 8192 \)). The \( c \) normalization constant in (1) was chosen so that \( R(\tau = 0) = 1 \). The autocorrelation time \( \tau_c \) of \( I(t) \) was determined from the condition \( R(\tau_c) = 1/\sqrt{2} \), and was found to be approximately 60 \( \mu \)s. This value of \( \tau_c \) was well reproducible when it was calculated from different recorded data sequences. Figure 2(b) shows the \( R(\tau) \) autocorrelation function of the current signal plotted in Fig. 1(a).

The phase space trajectory of the system was reconstructed using time delayed values of the discharge current \([18]\). The plot of the trajectory on the \( I(t + \tau), I(t) \) plane displayed in Fig. 3 suggests that the attractor is embedded in a higher-dimensional space \((D > 3)\). The time delay \( \tau \) used for the reconstruction of the trajectories was chosen to be equal to the autocorrelation time \( \tau_c = 60 \mu \)s.

The power density spectrum

\[
S(f) = |\mathcal{F}\{I(t)\}|^2
\]

of the current signal (\( \mathcal{F} \) denotes the Fourier transform) is plotted in Fig. 4. The \( S(f) \) spectrum was calculated by 8192 point FFT with a resolution \( \Delta f = 24.4 \) Hz. We have used 3-point adjacent averaging to smooth the spectrum. There is a wide range of frequencies where the \( S(f) \) spectrum can be well approximated by \( S(f) \propto 1/f^\alpha \) with a rather high exponent of \( \alpha \approx 4.3 \). It can be seen in Fig. 4 that the spectrum also contains a characteristic peak at \( f \approx 3 \) kHz and its subharmonic \((f/2)\). In the low-frequency part (below \( f \approx 3 \) kHz) the spectrum contains a considerable part of the power of the signal. In this frequency region the spectrum no longer scales as \( 1/f^\alpha \).

To obtain the correlation dimension of the recorded signal, the correlation sums of \( I(t) \)
Fig. 3. The $I(t+\tau)$ vs $I(t)$ phase space trajectory of the system reconstructed from a measured 8 kword long data sequence. The time delay $\tau$ equals the $\tau_c$ autocorrelation time of $I(t)$. (Same experimental conditions as in Fig. 2.)

Fig. 4. The $S(f)$ power density spectrum of current oscillations. The thick line is an apparent linear fit to the $S(f)$ spectrum having a slope of $-4.3$. (Same experimental conditions as in Fig. 2.)
were calculated for different embedding dimensions \( m \). We have used 16 kword long data sequences and have calculated the correlation sums using the method of Grassberger and Procaccia [19]:

\[
C^m(r) = \frac{1}{N^2} \sum_{i<j} \Theta(r - |\mathbf{X}_i - \mathbf{X}_j|),
\]

where \( \Theta \) is the Heaviside function and the \( m \) dimensional \( \mathbf{X} \) vectors consist of time delayed values of the current signal: \( \mathbf{X}_i = \{I(t_i), I(t_i + \tau), \ldots, I(t_i + (m-1)\tau)\} \). As can be seen in Fig. 5(a), the \( C^m(r) \) functions were found to scale as

\[
C(r) \propto r^\nu
\]

for a reasonably wide range of \( r \).

The correlation exponent \( \nu \) as a function of \( m \) (embedding dimension) is plotted in Fig. 5(b). It can be seen that \( \nu \) saturates with increasing embedding dimension proving the existence of a strange attractor. The \( \nu(m) \) function indicates a correlation dimension of \( D_2 \equiv 6.3 \). To test the estimation of the correlation dimension the correlation sums and the correlation exponents were also calculated for a 16 kword sequence of (artificially generated) white noise. The results of this calculation are also represented in Fig. 5(b).
DISCUSSION

The weakly ionized plasma of the glow discharge in addition to electrons contains a number of different atomic and molecular species, the most important being the atomic and molecular ions (\(\text{He}^+\), \(\text{He}_2^+\)) and metastable atoms (\(\text{He}^m\)). The concentrations of these ‘active’ plasma species are coupled to each other by collision processes and are also influenced by particle–wall interactions. We believe that the chaotic behaviour of the plasma basically originates in the spatio–temporal changes of the concentrations of these species.

The correlation dimension obtained from our experiment \((D_2 \approx 6.3)\) agrees surprisingly well with a theoretical value \((D \approx 6.31)\) of fully developed turbulence, calculated on the basis of a model independent, general theory [20–22]. Fully developed turbulence is considered here as a state of a space and time system, in the first moment, when non-periodic and non-quasiperiodic motion set on. We recall that our signal was recorded nearly at the moment of the onset of chaotic oscillations (very near to that pressure at which the chaotic oscillations appeared), in agreement with the above definition of fully developed turbulence. It is noted that our system is 4-dimensional (three spatial dimensions and time).

According to [23] the power spectrum of fluctuations of a 4-dimensional system has a scaling exponent \(\alpha = 5/3\) in the state of fully developed turbulence. The frequency spectrum calculated from our measured time series of the discharge current, plotted in Fig. 4 indicates that there exist a scaling region at high frequencies. For this region we obtained a scaling exponent \(\alpha = 4.3\). The reason of the disagreement between theoretical [23] and our (experimentally observed) scaling exponent is not yet understood. It is noted that the theory presented in [23] predicts two universal scaling exponents \(\alpha = 3/2\) and \(\alpha = 5/3\) for turbulent states. These values of the \(\alpha\) scaling exponent are found very frequently in completely different physical systems.

Dimensional analysis of turbulent signals was also applied for tokamak plasmas [9, 12]. These investigations gave different correlation dimensions between 2.6 and 3.2 [9] and between 6 and 16 [12], depending on experimental conditions. For glow discharges low correlation dimensions (between 2.3 and 3.0) were found in Ref. [8].

Experimentally observed signals were also analysed in terms of dimension and scaling exponents in the papers of Komori \(\text{et al.}\) [15] and Gyergyek \(\text{et al.}\) [17]. Komori \(\text{et al.}\) [15] have studied the plasma edge turbulence in a compact helical system. They have obtained correlation dimensions between \(D = 6\) and 8.4, and have observed \(1/f^\alpha\) power spectrum with \(\alpha \approx 2\). Gyergyek \(\text{et al.}\) [17] have investigated a weakly magnetized discharge plasma and obtained correlation dimensions between \(D = 1.2\) and 2. They have found the \(\alpha\) exponent to range between 3 and 6, depending on the experimental conditions. In both measurements the scaling region was at the high frequency part of the spectrum, similar to our measurements.

Acknowledgement — We thank Professor P. Szépfalussy and Dr K. Rózsa for useful discussions. The construction of the discharge tube by Ms T. J. Forgács, Mr J. Tóth, Mr E. Sárközi and Mr Gy. Császár and the support of the OTKA Hungarian Science Foundation (Grant No. F-7475) are gratefully acknowledged.

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