

Particle simulation methods for studies of low-pressure plasma sources

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W. Crookes Prize lecture

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Prof.W. Crookes and the 4th state of matter





- move from cathode to anode
- can be deflected by magnetic field
- 4th state of matter, consisting of negatively charged particles

THANKS : Colleagues, Coworkers & Friends



GLOW DISCHARGE PHYSICS

CHARGED PARTICLE KINETICS

M. Jánossy G. J. Kalman K. Rózsa K. I. Golden L. C. Pitchford L. Csillag B. Nyíri M. Bonitz A. V. Phelps J. Goree D. Marić S. Holló P. Hartmann A. Derzsi J. Schulze Z. Lj. Petrović G. Malović I. Korolov A. Bogaerts J. Karácsony H. R. Skullerud U. Czarnetzki D. Loffhagen E. Schüngel G. Bánó L. Szalai A. Gallagher J. P. Boeuf K. Kutasi J. Glosik K. Wiesemann F. Gordillo-Vazquez M. M. Turner S. Matejčik F. Sigeneger P. Simon L. Tsendin M. Piheiro J. Thomán-Forgács P. Horváth N. Sadeghi N. Pinhão B. Jelenković D. Luggenhölscher J. Tóth E. Sárközi Gy. Császár

RADIOFREQUENCY DISCHARGES

PHYSICS OF COMPLEX PLASMAS

GAS LASER RESEARCH

Particle simulation methods for studies of low-pressure plasma sources



(Diverse) startup thoughts

- * WHAT do we do? Describe motion and interactions of charged particles in order to understand the physics of gas discharges
- * WHY do we need simulations? Kinetic level, flexible, visualization
- * THIS TALK intends to illustrate the capabilities of particle simulations methods we start with very elementary examples and proceed towards more complex topics
- * "Bloom where you've been planted"

Topics

- * Charged particle kinetics in homogeneous field
- * Modeling of cold-cathode glow discharges
- ***** Particle in Cell simulation of radiofrequency discharges

(Few words about) Computational resources







I. Charged particle kinetics

- * Basics of Monte Carlo simulation
- * Velocity distribution functions and transport parameters in homogeneous field
- * Spatio-temporal relaxation of the electron gas
- * Gas breakdown

Monte Carlo methods





Monte Carlo methods: a simple example



Monte Carlo methods are used in all disciplines of science

How to approximate π ?



.... by the perimeters of polygons inscribed and circumscribed about a given circle (goes back to Archimedes) Monte Carlo approach:

Throw darts with uniform distribution on a square board



 $\frac{N(\text{red})}{N(\text{all})}$ $\frac{\pi}{4}$

Efficiency / computing speed A different way of thinking...

Monte Carlo method: particle transport



Transit of a particle through a slab of background gas



N background gas particles within the slab, each with same cross section

Area occupied by particles: $S = N\sigma$

Total area: A

Probability of collision:

$$dp(x) = \frac{S}{A} = \frac{N\sigma}{A} = \frac{(nAdx)\sigma}{A} = n\sigma dx$$
$$dp(t) = n\sigma v dt$$

For small time increments:

$$p(\Delta t) = n\sigma v\Delta t$$
$$\Delta t \to 0$$

In general:

$$p(\Delta t) = 1 - \exp\left[-\int_{0}^{\Delta t} n\sigma[v(t)]vdt\right]$$
$$p(\Delta s) = 1 - \exp\left[-\int_{0}^{\Delta s} n\sigma[v(t)]ds\right]$$

See (much) more, e.g.: S. Longo, PSST **15**, S181 (2006)

Monte Carlo method: particle transport





- Processes
 - elastic scattering,
 - electronic excitation,
 - ionization
 - attachment
- Chosen in a probabilistic manner:
 - free path,
 - type of collision,
 - new direction
 - 1 particle \rightarrow avalanche



Monte Carlo method: particle transport



- Acceleration techniques null-collision method [H. R. Skullerud, J. Phys. D 1, 1567 (1968)]
- Rescaling techniques very strong ionization / attachment [Y. M. Li, L. C. Pitchford, and T. J. Moratz, Appl. Phys. Lett. 54, 1403 (1989)]
- Cold gas approximation (collision partner is at rest)
 - usually valid for electrons, unless *E/n* is "very low" [M. Yousfi, A. Hennad, and A. Alkaa, Phys. Rev. E 49, 3264 (1994)]
 - at very low E/n, as well as for simulations of ion transport the collision partner must be chosen from a background gas with Maxwellian velocity distribution
- Monte Carlo simulation vs. Boltzmann equation
 - equivalent, both have their advantages [for comparison of different techniques see e.g. N. R.
 Pinhão, Z. Donkó, D. Loffhagen, M. J. Pinheiro and E. A. Richley, Plasma Sources Sci. Technol. 13, 719 (2004)].
 - *a lot of progress on BE solution* (multiterm methods, S. Dujko, ESCAMPIG 2010, also Greifswald group,)
- Benchmark your code! [A. M. Nolan, M. J. Brennan, K. F. Ness and A. B. Wedding, 1997, J. Phys. D 30, 2865; Z. M. Raspopović, S. Sakadzić, S. A. Bzenić, Z. Lj. Petrović, 1999, IEEE Trans. Plasma Science 27, 1241.]

Monte Carlo method - null-collision technique





MC simulation: electron avalanches





Electron velocity distribution in homogeneous electric field



Electron velocity distribution in homogeneous electric field



Spatio-temporal relaxation of an electron swarm





D. Loffhagen, R. Winkler, Z. Donkó, Eur. Phys. J. Appl. Phys. 18, 189 (2002).

(calculated with both MC simulation and Boltzmann equation)

Gas breakdown







Electron motion at left side of the Paschen curve becomes highly nonlocal → need for kinetic description Measured breakdown curves for helium gas:



What causes the strange shape for Helium ??

P. Hartmann, Z. Donkó, G. Bánó, L. Szalai, K. Rózsa, Plasma Sources Sci. Technol. **9**, 183 (2000).

Paschen curve in helium



Simulation including the Monte Carlo treatment of electrons, ions, as well as fast atoms

$e^- + X \to e^- + X$	Elastic collision
$e^- + X \rightarrow e^- + X^*$	Excitation
$e^- + X \rightarrow 2e^- + X^+$	Ionization
$\rm X^+ + \rm X \rightarrow \rm X^+ + \rm X^F$	Elastic collision (isotropic part)
$\rm X^+ + \rm X \rightarrow \rm X^+ + \rm X^F$	Elastic collision (backward part)
$X^+ + X \to X^+ + X^*$	Excitation
$X^+ + X \to 2X^+ + e^-$	Ionization
$X^F + X \to X^F + X^F$	Elastic collision
$\mathbf{X}^{\mathrm{F}} + \mathbf{X} \to \mathbf{X}^{\mathrm{F}} + \mathbf{X}^{*}$	Excitation
$\rm X^F + \rm X \rightarrow \rm X^F + \rm X^+ + e^-$	Ionization

+ reasonable data for secondary yields for all these species P. Hartmann, Z. Donkó, G. Bánó, L. Szalai, K. Rózsa, Plasma Sources Sci. Technol. **9**, 183 (2000).



What causes the strange shape for Helium ??

(i) ion-induced ionization,

(ii) secondary electron emission due to fast atoms



II. Modeling of cold-cathode glow discharges

- * Fluid models how far can we go without kinetic simulations?
- * Hybrid models ionization source calculated at kinetic level
- * Heavy-particle hybrid models
- * Applications

Self-consistent gas discharge models: a simple fluid model



Self-consistent = ???
What do we need for the mathematical description ?

Continuity equations:

 $\frac{\partial n_{\rm i}}{\partial t} + \frac{\partial \phi_{\rm i}}{\partial x} = S_{\rm i}$ $\frac{\partial n_{\rm e}}{\partial t} + \frac{\partial \phi_{\rm e}}{\partial x} = S_{\rm e}$

Momentum transfer:

$$\phi_{i} = \mu_{i}n_{i}E - \frac{\partial(n_{i}D_{i})}{\partial x}$$
$$\phi_{e} = -\mu_{e}n_{e}E - \frac{\partial(n_{e}D_{e})}{\partial x}$$
Poisson equation:

 $\Delta V = -\frac{e}{\varepsilon_0}(n_+ - n_-)$

Source functions

$$S_- = S_+ = \alpha \Phi_-$$

 $\alpha/n = f(E/n)$

local field approximation

Boundary conditions:

$$V(0) = 0, V(L) = V_0$$

$$\frac{\partial n_{\rm i}}{\partial x}\Big|_0 = 0 \qquad n_{\rm i}(L) = 0$$

$$n_{\rm e}(0)v_{\rm e}(0) = \gamma n_{\rm i}(0)v_{\rm i}(0)$$
$$n_{\rm e}(L) = 0$$



- Cold-cathode abnormal glow
- 1 dimension
- "<u>short discharge</u>": D >> L
- Radial losses can be neglected
- 1-dimensional model
- Processes:
 - Drift
 - Diffusion
 - Ionization

Results: simple fluid model



p = 40 Pa (Ar), V = 441 V, $T_g = 300$ K, L = 3 cm, $\gamma = 0.033$



Cathode sheath + negative glow structure reproduced Using E/n-dependent transport parameters is problematic \longrightarrow

Can we do better?

Z. Donkó, P. Hartmann, K. Kutasi, Plasma Sources Sci. Technol. 15, 178 (2006)W. J. M. Brok, personal communication

Extended fluid model



Energy equation & Ionization source

Use 3rd moment of BE; equation for energy density ($n_arepsilon=n_{
m e}ar{arepsilon}$): $\frac{\partial n_{\varepsilon}}{\partial t} + \frac{\partial \phi_{\varepsilon}}{\partial x} = S_{\varepsilon} - L_{\varepsilon}$ Energy source from field: Energy loss due to collisions: Energy flux: $L_{\varepsilon}(x) = k_L[\bar{\varepsilon}(x)]n_e(x)$ $S_{\varepsilon}(x) = -E(x)\phi_{\rm e}e$ $\phi_{\varepsilon} = -\mu_{\varepsilon} n_{\varepsilon} E - \frac{\partial (n_{\varepsilon} D_{\varepsilon})}{\partial r}$ Possibilities to calculate the ionization source:

Flux - based:

Rate coefficient - based:

$$S(x) = \alpha[\bar{\varepsilon}(x)]|\phi_{\rm e}(x)|$$
$$S(x) = k_{\rm i}[\bar{\varepsilon}(x)]n_{\rm e}(x)N$$

 $k_{\rm i}(\bar{\varepsilon}) = \sqrt{\frac{2e}{m}} \int_0^\infty \varepsilon \sigma_{\rm i}(\varepsilon) F_0(\varepsilon) \mathrm{d}\varepsilon$

see e.g. G J M Hagelaar and L C Pitchford, Plasma Sources Sci. Technol. 14, 722 (2005)

Extended fluid model



Transport coefficients & rates

To obtain these data: (1) carry out swarm calculations (MC) for a series of E/n values, and (2) organize the data as a function of the electron mean energy



Extended fluid model : results



Can we do better?





Most of the results are close to those of "simple" fluid models Extended fluid model with flux-based source calculation predicts a field reversal

BUT: still large differences between the results of different models

Hybrid model

The idea:

calculate the ionization source from kinetic simulation of fast electrons

Monte Carlo simulation - becomes inefficient for slow electrons - treat slow electrons as fluid

M. Surendra, D. B. Graves, and G. M. Jellum, Phys. Rev. A **41**, 1112 (1990).

J. P. Boeuf and L. C. Pitchford, IEEE Trans. Plasma Sci. **19**, 286 (1991).

A. Fiala, L. C. Pitchford, and J. P. Boeuf, Phys. Rev. E **49**, 5607 (1994).

A. Bogaerts, R. Gijbels, and W. J. Goedheer, J. Appl. Phys. 78, 2233 (1995).

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Hybrid model : results

Significant differences: e.g. orders of magnitude difference between the charge densities

"On the accuracy and limitations of fluid models of the cathode region of dc glow discharges" A. Derzsi, P. Hartmann, I. Korolov, J. Karácsony, G. Bánó, and Z Donkó, J. Phys. D: Appl. Phys. **42,** 225204 (2009)

Hybrid model : consistency (1)

Hybrid model : consistency (2)

MODELS

Simple fluid:

 $\dots kT_{e} = 0.1 \text{ eV}$

Extended fluid:

Hybrid:

 $-kT_{\rho} = 1 \text{ eV}$

 $-\mathsf{S}=\phi_{\rho}\,\alpha\,(\,\overline{\varepsilon}\,)$

 $-S = k_i N n_o$

kT_e = 0.1 eV

K. Rózsa, A. Gallagher and Z. Donkó: "Excitation of Ar lines in the cathode region of a DC discharge", Physical Review E **52**, 913-918 (1995)

EXPERIMENTS

It is only the hybrid model that predicts an exponentially falling ionization (and excitation) source

beyond the sheath-glow boundary

Ionization source

[10¹⁶ cm⁻³ s⁻¹]

10

10⁻²

10⁻³

10⁻⁴

10⁻⁵

D. Marić, K. Kutasi, G. Malović, Z. Donkó and Z. Lj. Petrović: "Axial emission profiles and apparent secondary electron yield in abnormal glow discharges in argon", Eur. Phys. J D **21**, 73 (2002)

Analysis of the distribution functions in the DC glow sheath

The spatial variation of the distribution function in the cathode sheath

Analysis of the VDF in the DC glow sheath

Application of hybrid models: (1) Townsend discharge – abnormal glow transition

Experiments: B. M. Jelenković and A. V. Phelps, J. Appl. Phys. **85**, 7089 (1999) Heavy particle hybrid model: Monte Carlo simulation of *fast electrons*, *fast positive ions* and *fast neutrals*

Heavy-particle hybrid model

Discharge transient: temporal changes of light intensity

Z. Donkó, J. Appl. Phys. 88, 2226 (2000)

B. M. Jelenković and A. V. Phelps, J. Appl. Phys. 85, 7089 (1999)

Application of hybrid models: (2) Heavy-particle excitation in the cathode region

Application of hybrid models: (2) Heavy-particle excitation in the cathode region

III. Radiofrequency discharges

- * Modeling basics of Particle-in-Cell method
- * Flux-energy distribution of ions at the electrodes
- Independent control of ion properties

Radiofrequency discharges

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Bounded plasma PIC simulation with Monte Carlo collisions (PIC/MCC)

Particle-in-Cell simulation of RF discharges

Particle-in-Cell simulation of RF discharges

Ar @ p = 50 mTorr, f = 13.56 MHz, V = 350 V

Particle-in-Cell simulation of RF discharges

Electric field Electron current Grounded 1.0 1.0 electrode 7.450E4 2.940E4 6.457E4 2.546E4 5.463E4 2.152E4 0.8 0.8 4.470E4 1.758E4 3.477E4 1.364E4 2.483E4 9700 0.6 0.6 -1.490E4 5760 7/x 0.4 4967 x/L 1820 -4967 -1.490E4 -2.483E4 -2120 0.4 -6060 -10000 -3.477E4 -1.394E4 -4.470E4 0.2 0.2 -1.788E4 -5.463E4 -2.182E4 -6.457E4 -2.576E4 -7.450E4 -2.970E4 0.0 0.0 -Powered 0.5 0.5 0.0 1.0 0.0 1.0 electrode Electron velocity Ionization source 1.0 1.0 1.860E21 6.250E7 1.705E21 RF 4.995E7 1.550E21 0.8 0.8 3.740E7 1.395E21 2.485E7 1.240E21 0.6 0.6 x/L 1.230E7 1.085E21 / L 9.300E20 -2.500E5 × 0.4 7.750E20 0.4 -1.280E7 6.200E20 -2.535E7 4.650E20 -3.790E7 0.2 0.2 -3.100E20 -5.045E7 1.550E20 -6.300E7 0.0 0.0 0.5 0.5 0.0 0.0 1.0 1.0 t/T t / T units in SI $V(t) = V_0 \sin(2\pi f t)$

Ar @ p = 50 mTorr, f = 13.56 MHz, V = 350 V

Flux-energy distributions of ions reaching the electrodes

Formation of ion energy distribution (single ion trajectories)

- * Ion transit time $\tau \gg T \rightarrow$ ions feel the average electric field
- * Periodic acceleration in the sheath
- * Effect of charge exchange collisions

Control of ion properties (flux and energy)

- P. C. Boyle, A. R. Ellingboe and M. M. Turner, Plasma Sources Sci. Technol. 13, 493-503, J. Phys. D. 37, 697 (2004)
- T. Kitajima, Y. Takeo, Z. Lj. Petrovic and T. Makabe, Appl. Phys. Lett. 77, 489 (2000)
- J. K. Lee, O. V. Manuilenko, N. Yu. Babaeva, H. C. Kim and J. W. Shon, Plasma Sources Sci. Technol. 14, 89 (2005).
- E. Kawamura, M. A. Lieberman and A. J. Lichtenberg, Phys. Plasmas 13 053506 (2006)
- M. M. Turner and P. Chabert, Phys. Rev. Lett. 96, 205001 (2006)
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- J. Schulze, Z. Donkó, D. Luggenhölscher and U. Czarnetzki, Plasma Sources Sci. Technol. 18, 034011 (2009)
- E. Semmler, P. Awakowicz and A. von Keudell, Plasma Sources Sci. Technol. 16, 839 (2007)
- A. Salabas and R. P. Brinkmann, Plasma Sources Sci. Technol. 14, 2 53 (2005)
- V. Georgieva and A. Bogaerts, Plasma Sources Sci. Technol., 15, 368 (2006)
- Z. Donkó and Z. Lj. Petrović, Jpn. J. Appl. Phys. 45, 8151 (2006)

Dual-frequency excitation: PIC results

Ar @ p = 25 mTorr, L = 2 cm HF = 60 V @ 100 MHz / LF = 200 V @ 10 MHz

Dual-frequency excitation: PIC results

Ar @ p = 25 mTorr, L = 2 cm HF = 60 V @ 100 MHz / LF = 200 V @ 10 MHz

Dual-frequency excitation: independent (?) control of ion properties

PIC vs. experiment

Spatio-temporal distribution of electron impact excitation

Alternative method: Electrical Asymmetry Effect

B. G. Heil, J. Schulze, T. Mussenbrock, R. P. Brinkmann and U. Czarnetzki 2008 IEEE Trans. Plasma Sci. **36** 1404

B. G. Heil, U. Czarnetzki, R. P. Brinkmann and T. Mussenbrock, 2008 J. Phys. D **41** 165202

 $\phi = V_0[\cos(\omega t + \theta) + \cos(2\omega t)]$

In geometrically symmetrical reactors a DC self bias builds up, if the discharge is driven by a waveform which contains an even harmonic of the fundamental frequency.

See much more: Uwe Czaretzki: "The Electrical Asymmetry Effect in Capacitive RF Discharges" [ESCAMPIG 2010]

Electrical asymmetry effect: flux-energy distributions of ions

Plasma Series Resonance

e.g.

U. Czarnetzki, T. Mussenbrock and R. P. Brinkmann, Phys. Plasmas 13, 123503 (2006).

T. Mussenbrock, R. P. Brinkmann, M. A. Lieberman, A. J. Lichtenberg, and E. Kawamura, Phys. Rev. Lett. **101** 085004 (2008).

PSR equation:

$$\Phi(t) = \Phi_{\text{sheath},1}(t) + \Phi_{\text{sheath},2}(t) + \Phi_{\text{Bulk}}(t)$$

special cases:

(1) strongly asymmetric,(2) symmetrical: sheaths cancel

Observation of PSR in a geometrically symmetric, electrically asymmetric discharge

 $\tilde{\phi} = V_0[\cos(\omega t + \theta) + \cos(2\omega t)]$

 $V_0 = 1000 V$, $\Theta = 7.5^{\circ}$, L = 2.5 cm, p = 3 Pa

Electron beams with a speed of $\approx 3 \times 10^6$ m/s.

Z. Donkó, J. Schulze, B. G. Heil and U. Czarnetzki, J. Phys. D: Appl. Phys. 42, 025205 (2009);

Z. Donkó, J. Schulze, U. Czarnetzki, D. Luggenhölscher, Appl. Phys. Lett. **94**, 131501 (2009).

Plasma Series Resonance

Observation of PSR oscillations in **electrically asymmetric** discharges

Z. Donkó, J. Schulze, B. G. Heil and U. Czarnetzki, J. Phys. D: Appl. Phys. 42, 025205 (2009); Z. Donkó, J. Schulze, U. Czarnetzki, D. Luggenhölscher, Appl. Phys. Lett. 94, 131501 (2009).

Velocity distribution functions in RF discharges

http://plasma.szfki.kfki.hu/~zoli/research/rf1/

Summary

- * This talk intended to illustrate the usefulness and applications of particle simulation methods in specific fields of gas discharge physics - from simple examples to more complicated systems:
 - * Charged particle kinetics in homogeneous field
 - Modeling of glow discharges
 - Particle in Cell simulation of radiofrequency discharges
- * Important: (i) connection with experiments, (ii) benchmarking, cross-checking

Thanks

COLLABORATORS, COLLEAGUES, FRIENDS,

..... TO ALL OF YOU FOR YOU ATTENTION