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Collective excitations in strongly coupled ultra-relativistic plasmas

Peter Hartmann[1,2], **Gabor J Kalman**[2], **Kenneth I Golden**[3] **and Zoltan Donko**[1,2]

[1] Research Institute for Solid State Physics and Optics of the Hungarian Academy of Sciences, H-1525 Budapest, PO Box 49, Hungary
[2] Department of Physics, Boston College, Chestnut Hill, MA 02467, USA
[3] Department of Mathematics and Statistics, College of Engineering and Mathematical Sciences, University of Vermont, Burlington, Vermont 05401-1455, USA

E-mail: kalman@bc.edu

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Abstract

In the collective mode spectrum of a relativistic, strongly coupled plasma, novel physical effects emerge, which are absent both in the weakly coupled relativistic and in the strongly coupled non-relativistic plasmas. Inspired by the pseudo-relativistic behavior of the electron gas in two-dimensional graphene layers, we address the problem of a classical two-dimensional, ultra-relativistic system of charged particles. We investigate the mode dispersion and damping both through molecular dynamics simulations and analytically via the quasi-localized charge approximation and develop modifications of the theory appropriate for this system. The new aspect introduced in the simulation is the decoupling of particle velocities from the particle momenta. As for new physical features, their origin is to be sought in the constancy of particle speeds and in the broad distribution of 'plasma frequencies', mimicking the similar distribution of momenta is causing the system to emulate the behavior of a collection of an infinite number of oscillators. Of particular interest is the strongly reduced damping at weak coupling, brought about by the disappearance of the Landau damping and the greatly enhanced damping at strong coupling, caused by the phase mixing of the coupled plasma oscillators. We suggest the possible experimental detection of these effects in graphene.

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The problem of relativistic plasmas first acquired attention in the works by Russian scientists [1]. The attention then focused on the behavior of plasmas where relativistic effects were due to high temperature. In a different line of research, the effect of Coulomb interaction on the behavior of the degenerate relativistic electron gas, principally in the context of white dwarf interiors, was studied in [2] and [3]: here relativistic effects are brought about by the high Fermi energy. In both situations, the Coulomb interaction energy is small compared to the kinetic energy, and in the usual parlance, the plasma is *weakly coupled.* In the case of an ultra-relativistic electron gas, when the ultra-relativistic limit $p_F/mc \to \infty$ and for the single particle energy $\varepsilon_p = c\sqrt{p^2 + (mc)^2}$, $\varepsilon_F = p_F/c$ (p_{F} is the Fermi momentum) are reached, this is, in fact, unavoidable, since the conventional coupling constant r_s goes to the limit $e^2/\hbar c \simeq 1/137$.

Relativistic effects manifest themselves in two different ways. First, once the particle velocities become comparable to c, the speed of light, the instantaneous static interaction between the charges has to be complemented by the fuller description through the retarded Lienard–Wiechert potential or by explicitly introducing photons as carriers of the interaction. Second, the relativistic particle dynamics has to be described in terms of a 'variable (momentum-dependent)' mass.

In this paper we address the behavior of the collective modes, in particular plasma oscillations under relativistic conditions. In the weak-coupling limit the two effects referred to the above are easily handled and do not lead to any dramatically new physical phenomena. As to the first, in the Vlasov (or random-phase-approximation (RPA)) description the interaction is mediated by the mean field only, which can easily be determined via a full set of Maxwell equations, as is already done in the conventional computation of the elements of the dielectric tensor $\varepsilon_{\mu\nu}(\mathbf{k}, \omega) = \delta_{\mu\nu} + \alpha_{\mu\nu}(\mathbf{k}, \omega)$. As to the second, the relativistically correct expression for the plasma frequency is obtained by the replacement of the particle mass by the (single-particle energy)/c^2: $m \Rightarrow \eta_p = \sqrt{p^2 + (mc)^2}/c$. As a result, each particle acquires its own 'plasma frequency' and the system becomes an aggregate of infinitely many components. However, within the RPA, this feature does not pose any particular problem, since in a multi-component system the individual polarizabilities simply add to generate the total longitudinal dielectric function $\varepsilon(\mathbf{k}, \omega) = 1 + \sum_A \alpha^A(\mathbf{k}, \omega)$, which is tantamount to replacing $1/m$ in the plasma frequency by a suitably defined $\langle 1/\eta_p \rangle$.

It is known, however, from the study of strongly coupled binary systems [4] that in such systems the simple addition law for the polarizabilities does not hold and the degeneracy of the plasma mode is removed. In fact, the combination of relativistic behavior and strong coupling creates a unique physical system, resembling that of a collection of a large number of coupled oscillators. Thus, in these novel systems the collective excitations are expected to exhibit an unconventional structure.

Recently, a number of plasma-like systems have become of interest, which are not weakly coupled and exhibit at the same time relativistic or quasi-relativistic behavior. Such systems are (i) laser-compressed plasmas, where the combination of high temperature and high density results in $\Gamma > 1$ [5], (ii) quark–gluon plasmas [6], where the dominant interaction is the Coulomb-like color field and where experiments seem to indicate a $\Gamma > 1$ situation and (iii) two-dimensional graphene layers [7], where the peculiar band structure leads to a pseudo-relativistic dynamics characterized by $m_{\mathrm{eff}} = 0$ and $\eta_p = p/c_{\mathrm{eff}}$ with $c_{\mathrm{eff}} \simeq c/400$ and $r_{s,\mathrm{eff}} \simeq 400/137$ (on a vacuum substrate).

In this paper, we explore the novel effects in the collective mode structure that arise in moderately to strongly coupled relativistic plasmas. The model we consider has been inspired by the electron gas in graphene. We have studied a 2D system of (negatively) charged classical pseudo-ultra-relativistic massless particles with single-particle energy $\varepsilon_{\mathbf{p}} = pc_{\mathrm{eff}}$, in a neutralizing background. We have investigated the collective mode structure analytically, by invoking the quasi-localized charge approximation (QLCA) (see, e.g., [8]), and we have performed a series of molecular dynamics (MD) simulations on the system over a wide range of coupling values. We contend that the classical approach is likely to provide an adequate

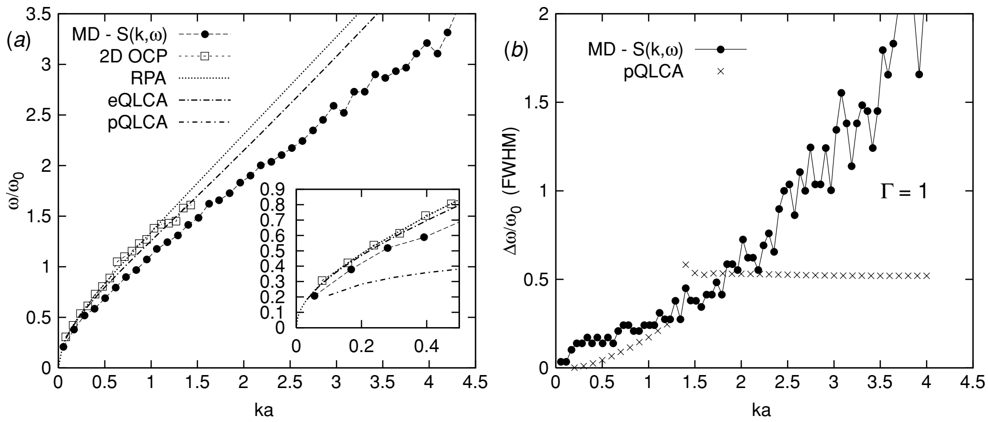

Figure 1. $\Gamma = 1$: (*a*) dispersion curves for from MD simulation and theoretical approaches; note that the OCP peak is detectable only up to $ka = 2$, while the current one survives for much higher *ka* values; (*b*) $S(\mathbf{k}, \omega)$ linewidth part from MD simulation and imaginary part of the pQLCA dispersion.

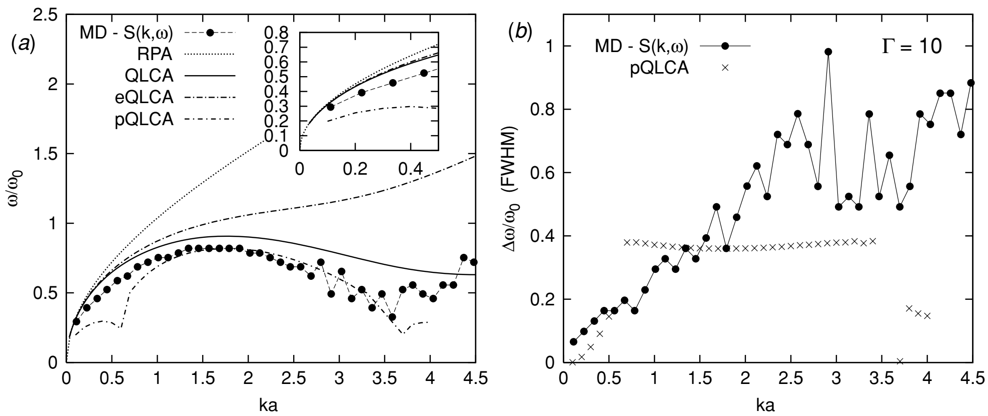

Figure 2. $\Gamma = 10$: (*a*) dispersion curves for from MD simulation and theoretical approaches; (*b*) $S(\mathbf{k}, \omega)$ linewidth from MD simulation and imaginary part of the pQLCA dispersion.

description for the collective mode structure of the degenerate quantum system as well; this contention is supported by the results of similar studies on a variety of systems [9]. The coupling constant for the 2D ultra-relativistic classical plasma and for the degenerate electron gas are defined, respectively, as $\Gamma = \beta e^2/a, r_s = e^2/\hbar c$ (c here and in the sequel stands for the effective 'light speed'; a is the Wigner–Seitz radius, $\pi n a^2 = 1$). Then, based on equating the average kinetic energies in the two systems, the correspondence $\Gamma \Rightarrow (3/2)\sqrt{g} r_s$ (g is the spin/valley multiplicity factor) can be established. The MD simulations are based on the PPPM technique [10]; particles are confined in a 2D layer with periodic boundary conditions. From the point of view of simulations, the most important new aspect is that momentum and velocity decouple from each other: the momentum satisfies the equation of motion, but the velocity determines the displacement. The absolute value of the velocity remains constant and interactions change its direction only.

Results of the MD simulation for the dispersion and damping of the plasmon mode for $\Gamma = 1$, 10 and 100 are displayed in figures 1–3, respectively. The dispersion is determined

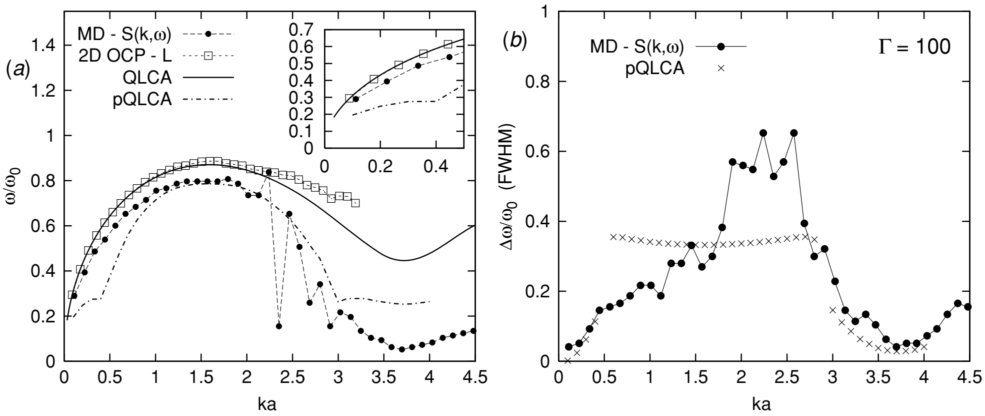

Figure 3. $\Gamma = 100$: (*a*) dispersion curves from MD simulation and theoretical approaches; (*b*) $S(\mathbf{k}, \omega)$ linewidth from MD simulation and imaginary part of the pQLCA dispersion.

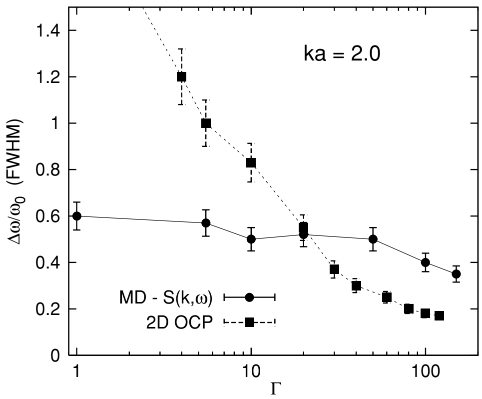

Figure 4. $S(\mathbf{k}, \omega)$ linewidth from MD simulations versus Γ. Compared are the results for classical 2D OCP and the present ultra-relativistic system at the wave number $ka = 2.0$.

from the peak positions of the dynamical structure function $S(\mathbf{k}, \omega)$, while the damping is characterized by the width of the respective peaks. For comparison, the corresponding results for the non-relativistic strongly coupled one-component plasma (OCP) are also shown. Figure 4 contrasts the damping as a function of Γ of the relativistic versus non-relativistic systems.

In order to describe the observed features various theoretical models have been explored. As a starting point, the classical longitudinal response function $\chi(\mathbf{k}, \omega)$ has been derived within the framework of the conventional Vlasov (RPA) description (this is the classical equivalent of the approach followed by Das Sarma *et al* [11]) and is used to generate the dispersion. It is important to realize that the phase velocity of the plasmon is always higher than c and consequently there is no Landau damping. The absence of the Landau damping is manifested in the structure of $\chi(\mathbf{k}, \omega)$ by its becoming purely real: remarkably, the absence of an imaginary part in $\chi(\mathbf{k}, \omega)$ allows one to express the dispersion relation explicitly (see equation (3)). The RPA description should be appropriate for $\Gamma \leqslant 1$. For higher Γ values the

QLCA theory, where the frequency is expressed in terms of the dynamical matrix $D(\mathbf{k})$,

$$D(\mathbf{k}) = \frac{1}{\bar{k}}\{\Lambda(0) - \Lambda(\mathbf{k})\}$$
$$\Lambda(\mathbf{k}) = \frac{1}{2\pi}\int \mathrm{d}^2\bar{r}\,\psi(\bar{\mathbf{r}})\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}h(r) \qquad \psi(\mathbf{r}) = \frac{1}{r^3}(3\cos^2\vartheta - 1) \tag{1}$$

($h(r)$ is the pair correlation function, $\bar{r} = r/a$ and $\cos\vartheta = \frac{\mathbf{k}\cdot\mathbf{r}}{kr}$) should be applicable. However, in contrast to the non-relativistic plasma, where it can be argued that for high enough coupling the effect of thermal velocities is negligible, here the $c =$ constant condition makes the validity of such an approximation doubtful. Thus, we have applied the 'extended' version of the QLCA, the eQLCA theory [12], where the random motion is accounted for by combining the RPA $\chi(\mathbf{k}, \omega)$ with the dynamical matrix $D(\mathbf{k})$. The resulting dispersion relation is

$$\frac{\Omega(\mathbf{k})}{\omega_0} = \sqrt{\bar{k}}\,\frac{1 + \frac{\bar{k}}{2\Gamma} + D(\mathbf{k})}{\sqrt{1 + \frac{\bar{k}}{4\Gamma} + D(\mathbf{k})}} \qquad \begin{aligned} &\bar{k} = ka \\ &\omega_0^2 = \frac{2\pi n e^2}{a\langle p/c\rangle} = \pi n e^2 c^2\beta/a = \pi n c^2\Gamma. \end{aligned} \tag{2}$$

It can be noted that equation (2) incorporates both the RPA and the 'classical' QLCA dispersion relations: the former is obtained by setting $D(\mathbf{k}) = 0$ and the latter by letting $\Gamma \to \infty$.

For sufficiently strong coupling, the physical effect that we have referred to as the system becoming a continuous distribution of coupled harmonic oscillators comes into play. Starting with the description of the system via a multi-component generalization of the QLCA [4], the model (to be referred to as pQLCA) leads to a set of N (=number of particles) coupled equations in momentum space; by converting the sum into an integral over the thermal distribution of momenta, an integral equation for $\Omega(\mathbf{k})$ is obtained:

$$-\frac{\Delta(\mathbf{k})}{\Delta(0)}\int_0^\infty \frac{\mathrm{e}^{-x}}{1 - \frac{[\Omega(\mathbf{k})]^2}{\Delta(0)}x}x\,\mathrm{d}x = 1 \qquad \begin{aligned} &\Delta(\mathbf{k}) = \omega_0^2\{\bar{k} + \Lambda(\mathbf{k})\} \\ &\Psi(\mathbf{r}) = \frac{1}{r^3}(3\cos^2\vartheta - 1). \end{aligned} \tag{3}$$

The mode frequency, $\Omega(\mathbf{k})$, that emerges as a solution is a complex quantity, indicating damped oscillations. Since the classical RPA is devoid of any damping mechanism, the origin of the damping here must be sought in the phase mixing of the distributed oscillators (cf [13]).

The dispersion curves resulting from the various approaches are displayed in figures 1–3 (the insets illustrate the behavior at low wave numbers).

The conclusions derived from the comparison between the theoretical predictions and the MD simulations can be summarized as follows:

(1) The low coupling $\Gamma = 1$. Figure 1 reveals that even at this low-coupling value the MD results deviate quite substantially from the RPA prediction, while the corresponding results for the non-relativistic (NR) OCP do not. This seems to indicate that strong-coupling effects are more pronounced in the relativistic than in the NR case. It can also be noted that the eQLCA provides an improvement over the RPA, coming closer to the MD points.

(2) The medium coupling $\Gamma = 10$. Figure 2 points at the complete inadequacy of the RPA in this domain. The best fit seems to be provided by the pQLCA; for small k-values, the MD results are below the QLCA and so are the pQLCA values, but with too much depression. Otherwise, the difference between the QLCA and the pQLCA is not substantial. Contrary to expectations, the eQLCA does not seem to provide an improvement at this Γ value.

(3) The high coupling $\Gamma = 100$ case (figure 3) further highlights (cf $\Gamma = 1$) the difference between the NR OCP and the relativistic system: this latter seems to be consistently below the NR OCP dispersion. It is the pQLCA that seems to best capture the qualitative features of the dispersion.

(4) Looking at the damping in figure 4, the difference between the Γ-dependences of the NR OCP and the relativistic system is striking. The absence of the Landau damping in the latter should be responsible for the much lower damping in the relativistic plasma at low Γ values. As to the high Γ domain, we believe that the broad phonon spectrum and the phase mixing of the distributed oscillators (cf above) is responsible for the enhanced damping in the relativistic plasma. This seems to be the explanation also for the fact that there is a non-vanishing damping even in the absence of Landau damping at any Γ value, as shown in the MD results of figures 1–3, which for small k values are reasonably matched by the pQLCA finding.

Commenting finally on the possible relevance of our results to graphene, we have already argued that the classical approach should render a reasonable description of the collective excitations even in the degenerate quantum situation. While the graphene r_s values are relatively modest ($c = 8 \times 10^5$ cm s^{-1}, $r_s = 2.74$ corresponds to $\Gamma = 8.22$), the behavior portrayed for $\Gamma = 1$ and $\Gamma = 10$ indicates that while strong-coupling effects would not be dramatic, they may be detectable by comparing future observations on graphene plasmon dispersion and damping with predictions of the RPA and of the current theory.

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